POCONO MOUNTAIN SCHOOL DISTRICT CURRICULUM

Honors Functions	Unit 1:	Functions and Graphs	Time Frame:	28 - 35 days

NATIONAL COMMON CORE STANDARDS:

Reason quantitatively and use units to solve problems

- **N.Q.1** Use units as a way to understand problems and to guide the solution of multi-step problems; choose and interpret units consistently in formulas; choose and interpret the scale and the origin in graphs and data displays.
- **N.Q.2** Define appropriate quantities for the purpose of descriptive modeling.
- **N.Q.3** Choose a level of accuracy appropriate to limitations on measurement when reporting quantities.

Creating equations that describe numbers or relationships

- **A.CED.1** Create equations and inequalities in one variable and use them to solve problems. *Include equations arising from linear and quadratic functions, and simple rational and exponential functions.*
- **A.CED.2** Create equations in two or more variables to represent relationships between quantities; graph equations on coordinate axes with labels and scales.
- **A.CED.3** Represent constraints by equations or inequalities, and by systems of equations and/or inequalities, and interpret solutions as viable or nonviable options in modeling context. *For example, represent inequalities describing nutritional and cost constraints on combinations of different foods.*
- **A.CED.4** Rearrange formulas to highlight a quantity of interest, using the same reasoning as in solving equations. *For example, rearrange Ohm's law V = IR to highlight resistance R.*

Understand solving equations as a process of reasoning and explain the reasoning

- **A.REI.1** Explain each step in solving a simple equation as following from the equality of numbers asserted at the previous step, starting from the assumption that the original equation has a solution. Construct a viable argument to justify a solution method.
- **A.REI.2** Solve simple rational and radical equations in one variable, and give examples showing how extraneous solutions may arise.

Solve equations and inequalities in one variable

- **A.REI.3** Solve liner equations and inequalities in one variable, including equations with coefficients represented by letters.
- **A.REI.4** Solve quadratic equations in one variable**.**
- **A.REI.4b** Solve quadratic equations by inspection (*e.g., for* $x^2 = 49$), taking square roots, completing the square, the quadratic formula and factoring, as appropriate to the initial form of the equation. Recognize when the quadratic formula gives complex solutions and write them as $a \pm bi$ for real numbers a and b.

Represent and solve equations and inequalities graphically

- **A.REI.10** Understand that the graph of an equation in two variables is the set of all its solutions plotted in the coordinate plane, often forming a curve (which could be a line).
- **A.REI.11** Explain why the *x-coordinates of the points where the graphs of the equations y = f(x) and y = g(x) intersect are the solutions of the equation f(x) = g(x); find the solutions approximately, e.g., using technology to*

MATHEMATICAL PRACTICES:

1. Make sense of problems and persevere in solving them.
2. Reason abstractly and quantitatively.
3. Construct viable arguments and critique the reasoning of others.
4. Model with mathematics.
5. Use appropriate tools strategically.
6. Attend to precision.
7. Look for and make use of structure.
8. Look for and express regularity in repeated reasoning.

graph the functions, make tables of values, or find successive approximations. Include cases where f(x) and/or g(x) are linear, polynomial, rational, absolute value, exponential, and logarithmic functions.

Understand the concept of a function and use function notation

- **F.IF.1** Understand that a function from one set (called the domain) to another set (called the range) assigns to each element of the domain exactly one element of the range. If *f* is a function and *x* is an element of its domain, then *f(x)* denotes the output of *f* corresponding to the input I x I The graph of *f* is the graph of the equation *y = f(x)*.
- **F.IF.2** Use function notation, evaluate functions for inputs in their domains, and interpret statements that use function notation in terms of a context.

Interpret functions that arise in applications in terms of the context

- **F.IF.4** For a function that models a relationship between two quantities interpret key features of graphs and tables in terms of the quantities, and sketch graphs showing key features given a verbal description of the relationship. Key features include: *intercepts; intervals where the function is increasing, decreasing, positive, or negative; relative maximums and minimums; symmetries; end behavior; and periodicity*.
- **F.IF.5** Relate the domain of a function to its graph and, where applicable, to the quantitative relationship it describes. *For examples, if the function h(n) gives the number person-hours it takes to assemble n engines in a factory, then the positive integers would be an appropriate domain for the function.*
- **F.IF.6** Calculate and interpret the average rate of change of a function (present symbolically or as a table) over a specified interval. Estimate the rate of change from a graph.

Analyze functions using different representations

- **F.IF.7** Graph functions expressed symbolically and show key features of the graph, by hand in simple cases and using technology for more complicated cases.
- **F.IF.7.a** Graph linear and quadratic functions and show intercepts, maxima, and minima.
- **F.IF.7.b** Graph square root, cube root, and piecewise-defined functions, including step functions and absolute value functions.
- **F.IF.7.c** Graph polynomial functions, identifying zeros when suitable factorizations are available, and showing end behavior.
- **F.IF.7.d** (+) Graph rational functions, identifying zeros and asymptotes when suitable factorizations are available, and showing end behavior.
- **F.IF.7.e** Graph exponential and logarithmic functions, showing intercepts and end behavior, and trigonometric functions, showing period, midline, and amplitude.

Build a function that models a relationship between two quantities

- **F.BF.1** Write a function that describes a relationship between two quantities.
- **F.BF.1.a** Determine an explicit expression, a recursive process, or steps for calculation from a context.
- **F.BF.1.b** Combine standard function types using arithmetic operations. *For example, build a function that models the temperature of a cooling body by adding a constant function to a decaying exponential, and relate these functions to the model.*
- **F.BF.1.c** (+) Compose functions. *For example, if T(y) is the temperature in the atmosphere as a function of height, and h(t) is the height of a weather balloon as a function of time, then T(h(t)) is the temperature at the*

location of the weather balloon as a function of time.

Build new functions from existing functions

- **F.BF.3** Identify the effect on the graph of replacing f(x) by f(x) + k, k f(x), f(kx), and f(x+k) for specific values of k (both positive and negative); find the value of k given the graphs. Experiment with cases and illustrate an explanation of the effects on the graph using technology. *Include recognizing even and odd functions from their graphs and algebraic expressions for them.*
- **F.BF.4** Find inverse functions.
- **F.BF.4.a** Solve and equation of the form f(x) = c for a simple function f that has an inverse and write an expression for the inverse. *For example, f(x) =2 x^3 or f(x) = (x+1)/(x-1) for x≠ 1.*
- **F.BF.4.b** (+) Verify by composition that on function is the inverse of another.
- **F.BF.4.c** (+) Read values of an inverse function from a graph or a table, given that the function has an inverse.
- **F.BF.4.d** (+) Produce an invertible function from a non-invertible function by restricting the domain.

ESSENTIAL QUESTIONS	VOCABULARY	ASSESSMENT
1. How would you use algebraic, numerical and graphical models to solve problems? 2. How would you analyze the characteristics of the basic functions? 3. Which basic functions would you use to build new functions? 4. Define functions and relations parametrically. 5. Find the inverse of a relation or function. 6. How would you investigate transformations of functions and parametric relations? 7. Which concepts of functions would you use in real world situations?	• Relation • Function • Inverse of a function • Parametric relations • Transformations	• Observation and questioning • Presentations and discussions • Projects and investigations • Mathematical writing • Homework • Quizzes • Tests

UNIT OF INSTRUCTION: Functions and Graphs	PA CORE STANDARDS	
	CC.2.1.HS.F.4 Use units as a way to understand problems and to guide the solution of multi-step problems. **CC.2.2.HS.D.2** Write expressions in equivalent forms to solve problems. **CC.2.2.HS.D.3** Extend the knowledge of arithmetic operations and apply to polynomials. **CC.2.2.HS.D.8** Apply inverse operations to solve equations or formulas for a given variable. **CC.2.2.HS.D.9** Use reasoning to solve equations and justify the solution method. **CC.2.2.HS.D.10** Represent, solve and interpret equations/inequalities and systems of equations/inequalities algebraically and graphically. **CC.2.2.HS.C.1** Use the concept and notation of functions to interpret and apply them in terms of their context. **CC.2.2.HS.C.2** Graph and analyze functions and use their properties to make connections between the different representations. **CC.2.2.HS.C.4** Interpret the effects transformations have on functions and find the inverses of functions. **CC.2.2.HS.C.5** Construct and compare linear, quadratic and exponential models to solve problems. **CC.2.2.HS.C.6** Interpret functions in terms of the situation they model. **Essential Understandings/Learning Activities:** Activity: Mathematical Definitions: Precalculus 1. Represent problems using different models 2. Fit curves to data 3. Solve equations algebraically Activity: Ten Commandment of Mathematics Activity: Algebra Card Tricks 4. Investigate graph failure 5. Examine the graphical representations of functions 6. Determine the domain and range of a function 7. Investigate the continuity of functions 8. Identify local extrema 9. Find the asymptotes of a function 10. Identify and analyze the twelve basic functions 11. Add, subtract, multiply, and divide functions 12. Find the composition of functions 13. Determine the domain of a composition 14. Use implicitly defined functions to define relations 15. Investigate parametric equations Activity: Crashing Airplanes 16. Find the inverse of a relation or function 17. Use the horizontal line test to determine if a relation has an inverse 18. Determine whether a function is one-to-one 19. Find equations for translations, reflections, stretches and shrinks of functions	

20. Examine combinations of transformations and the consequences of the order in which they are applied

Activity: Examining How Mathematics is Used in the workplace
Activity: The Point of No Return
Lab Activity: It Averages Out in the End

NOTE: Honors level students are expected to work on additional rigorous, challenging problems, proofs and application of concepts/skills as part of the course. Increased pace of instruction will occur.

DIFFERENTIATION ACTIVITIES:
Teacher directed differentiated instructional projects and activities are ongoing and based on student need.

ENRICHMENT:

- Red-Haired Older Son
- Project: Collaborative Investigation – Babylonian Square Roots
- Web-based Math Resources
- Small group instruction
- Teacher generated/differentiated instruction enrichment and activities
- Supporting the range of learners as per teacher manual
- Encourage and support learners in explaining how they applied their skills during mathematical tasks
- Precalculus teacher's resources and materials

REMEDIATION:

- Slope and Y-Intercept 1
- X- and Y-Intercepts
- Slope and Y-Intercept 2
- Graphing quadratics 1, 2, and 3
- Quick Graphs of Quadratic Equations
- Manipulating Powers (2 pages)
- Evaluating Rational Exponents
- Simplifying Radicals
- Domain and Range #1
- Domain and Range #2
- Sum, Difference, and Product
- Conjugate and Quotient

RESOURCES:

- Precalculus: Graphing, Numerical, Algebraic, 7th ed., Demana et. Al., ©2007
- http://www.sosmath.com/index/html
- www.algebrahelp.com
- www.coolmath.com
- www.mathleague.com
- www.interactmath.com

POCONO MOUNTAIN SCHOOL DISTRICT CURRICULUM

Honors Functions	Unit 2:	Polynomial, Power and Rational Functions	Time Frame:	38 days

NATIONAL COMMON CORE STANDARDS:

Extend the properties of exponents to rational exponents

- **N.RN.1** Explain how the definition of the meaning of rational exponents follows from extending the properties of integer exponents to those values, allowing for a notation for radicals in terms of rational exponents. *For example, we define* $(5^{1/3})^3$ to hold, so $(5^{1/3})^3$ *must equal 5.*
- **N.RN.2** Rewrite expressions involving radicals and rational exponents using the properties of exponents.

Use properties of rational and irrational numbers

- **N.RN.3** Explain why the sum of product of two rational numbers is rational; that the sum of a rational number and an irrational number is irrational; and that the product of a nonzero rational number and an irrational number is irrational.

Reason quantitatively and use units to solve problems

- **N.Q.1** Use units as a way to understand problems and to guide the solution of multi-step problems; choose and interpret units consistently in formulas; choose and interpret the scale and the origin in graphs and data displays.
- **N.Q.2** Define appropriate quantities for the purpose of descriptive modeling.
- **N.Q.3** Choose a level of accuracy appropriate to limitations on measurement when reporting quantities.

Perform arithmetic operations with complex numbers

- **N.CN.1** Use units as a way to understand problems and to guide the solution of multi-step problems; choose and interpret units consistently in formulas; choose and interpret the scale and the origin in graphs and data displays.
- **N.CN.2** Define appropriate quantities for the purpose of descriptive modeling.
- **N.CN.3** (+) Find the conjugate of a complex number; use conjugates to find moduli and quotients complex numbers.

Use complex numbers in polynomial identities and equations

- **N.CN.7** Solve quadratic equations with real coefficients that have complex solutions
- **N.CN.8** (+) Extend polynomial identities to the complex numbers. *For example, rewrite* $x^2 + 4$ *as* $(x + 2i)(x-2i)$.
- **N.CN.9** (+) Know the Fundamental Theorem of Algebra; show that it is true for quadratic polynomials.

Interpret the structure of expressions

- **A.SSE.1** Interpret expressions that represent a quantity in terms of its context.
- **A.SSE.1.a** Interpret parts of an expression, such as terms, factors, and coefficients.
- **A.SSE.1.b** Interpret complicated expressions by viewing one or more of their parts as a single entity. *For example, interpret* $P(1+r)^n$ *as the product of P and a factor not depending on P.*
- **A.SSE.2** Use the structure of an expression to identify ways to rewrite it. *For example, see* x^4-y^4 *as* $(x^2)^2 – (y^2)^2$, *thus recognizing it as a difference of squares the can be factored as* $(x^2-y^2)(x^2+y^2)$.

Write expressions in equivalent forms to solve problems

MATHEMATICAL PRACTICES:

1. Make sense of problems and persevere in solving them.
2. Reason abstractly and quantitatively.
3. Construct viable arguments and critique the reasoning of others.
4. Model with mathematics.
5. Use appropriate tools strategically.
6. Attend to precision.
7. Look for and make use of structure.
8. Look for and express regularity in repeated reasoning.

- **A.SSE.3** Choose and produce an equivalent form of an expression to reveal and explain properties of the quantity represented by the expression.
- **A.SSE.3.a** Factor a quadratic expression to reveal the zeros of the function it defines.
- **A.SSE.3.b** Complete the square in a quadratic expression to reveal the maximum or minimum value of the function it defines.

Understand the relationship between zeros and factors of polynomials

- **A.APR.2** Know and apply the Remainder Theorem: For a polynomial p(x) and a number a, the remainder on division by x – a is p(a), so p(a) = O if and only if (x – a) is a factor of p(x)
- **A.APR.3** Identify zeros of polynomials when suitable factorizations are available, and use the zeros to construct a rough graph of the function defined by the polynomial.

Use polynomial identities to solve problems

- **A.APR.4** Prove polynomial identities and use them to describe numerical relationships. *For example, the polyno9mial identity* $(x^2 + y^2)^2=(x^2 – y^2)^2 + (2xy)^2$ *can be used to generate Pythagorean triples.*

Rewrite rational expressions

- **A.PR.6** Rewrite simple rational expressions in different forms; write a(x)/b(x) in the form q(x) + r(x)/b(x), where a(x), b(x), q(x), and r(x) are polynomials with the degree of r(x) less than the degree of b(x), using inspection, long division, or, for the more complicated examples, a computer algebra system.
- **A.PR.7** (+)Understand that rational expressions form a system analogous to the rational numbers, closed under addition, subtraction, multiplication, and division by a nonzero rational expression; add, subtract, multiply, and divide rational expressions.

Create equations that describe numbers or relationships

- **A.CED.1** Create equations and inequalities in one variable and use them to solve problems. *Include equations arising from linear and quadratic functions, and simple rational and exponential functions.*
- **A.CED.2** Create equations in two or more variables to represent relationships between quantities; graph equations on coordinate axes with labels and scales.
- **A.CED.3** Represent constraints by equations or inequalities, and by systems of equations and/or inequalities, and interpret solutions as viable or nonviable options in modeling context. *For example, represent inequalities describing nutritional and cost constraints on combinations of different foods.*

Understand solving equations as a process of reasoning and explain the reasoning

- **A.REI.1** Explain each step in solving a simple equation as following from the equality of numbers asserted at the previous step, starting from the assumption that the original equation has a solution. Construct a viable argument to justify a solution method.
- **A.REI.2** Solve simple rational and radical equations in one variable, and give examples showing how extraneous solutions may arise.

Solve equations and inequalities in one variable

- **A.REI.3** Solve liner equations and inequalities in one variable, including equations with coefficients represented by letters.
- **A.REI.4** Solve quadratic equations in one variable.

- **A.REI.4.a** Use the method of completing the square to transform any quadratic equation in x into an equation of the form $(x - p)^2 = q$ that has the same solutions. Derive the quadratic formula from this form.
- **A.REI.4b** Solve quadratic equations by inspection (*e.g., for* $x^2 = 49$), taking square roots, completing the square, the quadratic formula and factoring, as appropriate to the initial form of the equation. Recognize when the quadratic formula gives complex solutions and write them as $a \pm bi$ for real numbers a and b.

Represent and solve equations and inequalities graphically

- **A.REI.10** Understand that the graph of an equation in two variables is the set of all its solutions plotted in the coordinate plane, often forming a curve (which could be a line).
- **A.REI.11** Explain why the *x-coordinates of the points where the graphs of the equations y = f(x) and y = g(x) intersect are the solutions of the equation f(x) = g(x); find the solutions approximately, e.g., using technology to graph the functions, make tables of values, or find successive approximations. Include cases where f(x) and/or g(x) are linear, polynomial, rational,, absolute value, exponential, and logarithmic functions.*

Interpret functions that arise in applications in terms of the context

- **F.IF.4** For a function that models a relationship between two quantities interpret key features of graphs and tables in terms of the quantities, and sketch graphs showing key features given a verbal description of the relationship. Key features include: *intercepts; intervals where the function is increasing, decreasing, positive, or negative; relative maximums and minimums; symmetries; end behavior; and periodicity.*
- **F.IF.5** Relate the domain of a function to its graph and, where applicable, to the quantitative relationship it describes. *For examples, if the function h(n) gives the number person-hours it takes to assemble n engines in a factory, then the positive integers would be an appropriate domain for the function.*
- **F.IF.6** Calculate and interpret the average rate of change of a function (present symbolically or as a table) over a specified interval. Estimate the rate of change from a graph.

Analyze functions using different representations

- **F.IF.7** Graph functions expressed symbolically and show key features of the graph, by hand in simple cases and using technology for more complicated cases.
- **F.IF.7.a** Graph linear and quadratic functions and show intercepts, maxima, and minima.
- **F.IF.7.b** Graph square root, cube root, and piecewise-defined functions, including step functions and absolute value functions.
- **F.IF.7.c** Graph polynomial functions, identifying zeros when suitable factorizations are available, and showing end behavior.
- **F.IF.7.d** (+) Graph rational functions, identifying zeros and asymptotes when suitable factorizations are available, and showing end behavior.
- **F.IF.7.e** Graph exponential and logarithmic functions, showing intercepts and end behavior, and trigonometric functions, showing period, midline, and amplitude.
- **F.IF.8** Write a function defined by an expression in different but equivalent forms to reveal and explain different properties of the function
- **F.IF.8.a** Use the process of factoring and completing the square in a quadratic function to show zeros, extreme values, and symmetry of the graph, and interpret these in terms of a context.
- **F.IF.9** Compare properties of two functions each represented in a different way (algebraically, graphically, numerically in tables, or by verbal descriptions). *For example, given a graph of one quadratic function and an*

algebraic expression for another, say which has the larger maximum.

ESSENTIAL QUESTIONS	VOCABULARY	ASSESSMENT
1. How would you graph polynomial functions? 2. How would you investigate the power of functions? 3. How would you predict end behavior and determine the real zeros of polynomial functions? 4. Would you be able to determine rational zeros using the rational roots theorem, the factor theorem and synthetic division? 5. How would you investigate complex zeros? 6. What graphs would you use to analyze rational functions? 7. Which graphical and algebraic techniques would you use to solve equations and inequalities?	• Asymptote • Rational and Irrational Numbers • Polynomial Functions • Rational Functions • Power Functions • Rational Roots Theorem • Factor Theorem • Complex Number • Conjugates	• Observation and questioning • Presentations and discussions • Projects and investigations • Mathematical writing • Homework • Quizzes • Tests

UNIT OF INSTRUCTION: Polynomial, Power, and Rational Functions

PA CORE STANDARDS

CC.2.1.HS.F.1 Apply and extend the properties of exponents to solve problems with rational exponents.
CC.2.1.HS.F.2 Apply properties of rational and irrational numbers to solve real world or mathematical problems.
CC.2.1.HS.F.3 Apply quantitative reasoning to choose and interpret units and scales in formulas, graphs and data displays.
CC.2.1.HS.F.4 Use units as a way to understand problems and to guide the solution of multi-step problems.
CC.2.1.HS.F.5 Choose a level of accuracy appropriate to limitations on measurement when reporting quantities.
CC.2.1.HS.F.6 Extend the knowledge of arithmetic operations and apply to complex numbers.
CC.2.1.HS.F.7 Apply concepts of complex numbers in polynomial identities and quadratic equations.
CC.2.2.HS.D.1 Interpret the structure of expressions to represent a quantity of terms of its context.
CC.2.2.HS.D.2 Write expressions in equivalent forms to solve problems.
CC.2.2.HS.D.3 Extend the knowledge of arithmetic operations and apply to polynomials.
CC.2.2.HS.D.4 Understand the relationship between zeros and factors of polynomials to make generalizations about functions and their graphs.
CC.2.2.HS.D.5 Use polynomial identities to solve problems.
CC.2.2.HS.D.6 Extend the knowledge of rational functions to rewrite in equivalent forms.
CC.2.2.HS.D.7 Create the graph equations or inequalities to describe numbers or relationships.
CC.2.2.HS.D.8 Apply inverse operations to solve equations or formulas for a given variable.
CC.2.2.HS.D.9 Use reasoning to solve equations and justify the solution method.
CC.2.2.HS.D.10 Represent, solve and interpret equations/inequalities and systems of equations/inequalities algebraically and graphically.
CC.2.2.HS.C.1 Use the concept and notation of functions to interpret and apply them in terms of their context.
CC.2.2.HS.C.2 Graph and analyze functions and use their properties to make connections between the different representations.

CC.2.2.HS.C.3 Write functions or sequences that model relationships between two quantities.
CC.2.2.HS.C.6 Interpret functions in terms of the situation they model.

Essential Understandings/Learning Activities:

1. Graph polynomial functions of various degrees and investigate the shapes of the graphs
2. Find the rate of change of a function
3. Use regression models to solve problems
4. Determine the vertex and axis of symmetry for a quadratic function
5. Determine local maximum and minimum values
6. Analyze the characteristics of power and function
7. Use power functions to model real life problems
8. Investigate the end behavior of polynomial functions of even and odd degree
9. Find the end behavior model
10. Determine the zeroes of a polynomial function
11. Establish the multiplicity of the zeroes of a polynomial function
12. Explore the intermediate value property algebraically and graphically
13. Explore the division algorithm for polynomials
14. Use the remainder and factor theorem to test for zeroes
15. Use synthetic division as an aid to test for rational zeroes
16. Test for rational zeroes using the rational roots theorem
17. Determine the upper and lower bounds for real zeroes
18. Model real-world situations using polynomial functions
19. Perform operations with complex numbers
20. Determine complex zeroes
21. Investigate polynomial functions of even and odd degree and their possible number of real and complex zeroes
22. Find the domain of a rational function
23. Determine the asymptotes of a rational function
24. Graph rational functions
25. Solve rational equations
26. Determine when a rational equation has an extraneous solution
27. Use rational functions to solve real world problems

Activity: Designing a Juice Can Solve polynomial and rational inequalities, algebraically and graphically. Use a sign chart to solve inequalities

NOTE: Honors level students are expected to work on additional rigorous, challenging problems, proofs and application of concepts/skills as part of the course. Increased pace of instruction will occur.

POCONO MOUNTAIN SCHOOL DISTRICT CURRICULUM

<table>
<tr><td colspan="4">DIFFERENTIATION ACTIVITIES:
Teacher directed differentiated instructional projects and activities are ongoing and based on student need.</td></tr>
<tr><td>ENRICHMENT:</td><td colspan="2">Einstein's Problem
Web-based Math Resources
Small group instruction
Teacher generated/differentiated instruction enrichment and activities
Supporting the range of learners as per teacher manual
Encourage and support learners in explaining how they applied their skills during mathematical tasks
Precalculus teacher's resources and materials</td><td>REMEDIATION:</td><td>Synthetic Substitution
Synthetic Substitution (cont.)
The Remainder Theorem
The Factor Theorem
Dividing Polynomials
Synthetic Division</td></tr>
<tr><td>RESOURCES:</td><td colspan="4">Precalculus: Graphing, Numerical, Algebraic, 7th ed., Demana et. Al., ©2007
http://www.sosmath.com/index/html
www.algebrahelp.com
www.coolmath.com
www.mathleague.com
www.interactmath.com</td></tr>
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POCONO MOUNTAIN SCHOOL DISTRICT CURRICULUM

Honors Functions	Unit 3:	Exponential and Logarithmic Functions	Time Frame:	25 days

NATIONAL COMMON CORE STANDARDS:

Extend the properties of exponents to rational exponents

- **N.RN.1** Explain how the definition of the meaning of rational exponents follows from extending the properties of integer exponents to those values, allowing for a notation for radicals in terms of rational exponents. *For example, we define* $(5^{1/3})^3$ to hold, so $(5^{1/3})^3$ *must equal 5.*
- **N.RN.2** Rewrite expressions involving radicals and rational exponents using the properties of exponents.

Interpret the structure of expression

- **A.SSE.1** Interpret expressions that represent a quantity in terms of its context.
- **A.SSE.1.a** Interpret parts of an expression, such as terms, factors, and coefficients.
- **A.SSE.1.b** Interpret complicated expressions by viewing one and more of their parts as a single entity.
- **A.SSE.2** Use the structure of an expression to identify ways to rewrite it. *For example, see* $x^4 - y^4$ *as* $(x^2)^2 - (y^2)^2$, *thus recognizing it as a difference of squares that can be factored as* $(x^2 - y^2)(x^2 + y^2)$.

Write expressions in equivalent forms to solve problems

- **A.SSE. 3** Choose and produce an equivalent form of an expression to reveal and explain properties of the quantity represented by the expression.
- **A.SSE.3.a** Factor a quadratic expression to reveal the zeros of the function it defines.
- **A.SSE.3.c** Use the properties of exponents to transform expressions for exponential function. *For example, the expression* 1.15^t *can be rewritten as* $(1.15^{1/12})^{12t} \approx 1.012^{12t}$ *to reveal the approximate equivalent monthly interest rate if the annual rate is 15%.*

Create equations that describe numbers or relationships

- **A.CED.1** Create equations and inequalities in one variable and use them to solve problems. Include equations arising from linear and quadratic functions, and simple rational and exponential functions.
- **A.CED.2** Create equations in two or more variables to represent relationships between quantities; graph equations on coordinate axes with labels and scales.
- **A.CED.3** Represent constraints by equations and inequalities, and by systems of equations and/or inequalities, and interpret solutions as viable or nonviable options in a modeling context. *For example, represent inequalities describing nutritional and cost constraints on combinations of different foods.*
- **A.CED.4** Rearrange formulas to highlight a quantity of interest, using the same reasoning as in solving equations *For example, rearrange Ohm's law V = IR to highlight resistance.*

Analyze functions using different representations

- **F.IF.7** Graph functions expressed symbolically and show key features of the graph, by hand in simple cases and using technology for more complicated cases.
- **F.IF.7.e** Graph exponential and logarithmic functions, showing intercepts and end behavior, and trigonometric functions, showing period, midline, and amplitude.

MATHEMATICAL PRACTICES:

1. Make sense of problems and persevere in solving them.
2. Reason abstractly and quantitatively.
3. Construct viable arguments and critique the reasoning of others.
4. Model with mathematics.
5. Use appropriate tools strategically.
6. Attend to precision.
7. Look for and make use of structure.
8. Look for and express regularity in repeated reasoning.

- **F.IF.8** Write a function defined by an expression in different but equivalent forms to reveal and explain different properties of the function.
- **F.IF.8.b** Use the properties of exponents to interpret expressions for exponential functions. *For example, identify percent rate of change in functions such as* $y = (1.02)^t$, $y = (0.97)^t$, $y = (1.01)^{12t}$, *and* $y = (1.2)^{t/10}$, *and classify them as representing exponential growth or decay.*
- **F.IF.9** Compare properties of two functions each represented in a different way (algebraically, graphically, numerically in tables, or by verbal descriptions). *For example, given a graph of one quadratic function and an algebraic expression for another, say which has the larger maximum.*

Building functions

- **F.BF.4** Find inverse functions.
- **F.BF.4.a** Solve an equation of the form *f(x)=c* for a simple function *f* that has an inverse and write an expression for the inverse. *For example,* $f(x) = 2x^3$ *or* $fx = (x + 1)/(x – 1)$ for $x \neq 1$.
- **F.BF.4.b** (+) *Verify by composition that one function is the inverse for another.*
- **F.BF.4.c** (+) Read values of an inverse function from a graph or a table, given that the function has an inverse.
- **F.BF.4.d** (+) Produce an invertible function from a non-invertible function by restricting the domain.
- **F.BF.5** (+) Understand the inverse relationship between exponents and logarithms and use this relationship to solve problems involving logarithms and exponents.

Construct and compare linear, quadratic, and exponential models and solve problems

- **F.LE.1** Distinguish between situations that can be modeled with linear functions and with exponential functions.
- **F.LE.1.a** Prove that linear functions grow by equal differences over equal intervals, and that exponential functions grow by equal factors over equal intervals.
- **F.LE.1.b** Recognize situations in which one quantity changes at a constant rate per unit interval relative to another.
- **F.LE.1.c** Recognize situations in which a quantity grows or decays by a constant percent rate per unit interval relative to another.
- **F.LE.2** Construct linear and exponential functions, including arithmetic and geometric sequences, given a graph, a description of a relationship, or two input-output pairs (include reading these from a table).
- **F.LE.3** Observe using graphs and tables that a quantity increasing exponentially eventually exceeds a quantity increasing linearly, quadratically, or (more generally) as polynomial function.
- **F.LE.4** For exponential models, express as a logarithm the solution to ab^{ct} = where a, c, and d are numbers and the base b is 2, 10, or e; evaluate the logarithm using technology.

POCONO MOUNTAIN SCHOOL DISTRICT CURRICULUM

ESSENTIAL QUESTIONS	VOCABULARY	ASSESSMENT
1. How would you evaluate exponential expressions and graph exponential functions? 2. How would you use exponential growth and decay to model real-life problems? 3. How would you evaluate and graph common and natural logarithms? 4. How would you apply the properties of logarithms to solve exponential and logarithmic equations algebraically? 5. Show how to solve a variety of application problems involving logarithms and use exponential functions to solve business and finance applications.	• Exponential growth • Exponential decay • Logarithms	• Observation and questioning • Presentations and discussions • Projects and investigations • Mathematical writing • Homework • Quizzes • Tests

UNIT OF INSTRUCTION: Exponential and Logarithmic Functions	PA CORE STANDARDS	
	CC.2.1.HS.F.1 Apply and extend the properties of exponents to solve problems with rational exponents. **CC.2.2.HS.D.2** Write expressions in equivalent forms to solve problems. **CC.2.2.HS.D.8** Apply inverse operations to solve equations or formulas for a given variable. **CC.2.2.HS.C.2** Graph and analyze functions and use their properties to make connections between the different representation. **CC.2.2.HS.C.5** Construct and compare linear, quadratic and exponential models to solve problems. **CC.2.2.HS.C.6** Interpret functions in terms of the situation they model. **Essential Understandings/Learning Activities:** **1.** Graph exponential functions **2.** Perform transformations of the graphs of exponential functions **3.** Investigate the natural exponential functions **4.** Apply exponential functions to the real-world situations of growth and decay Activity: the M&M Function Activity Carbon Dating **5.** Investigate the inverse of the exponential function **6.** Graph the logarithmic function **7.** Change functions from exponential to logarithmic form Activity: Logarithmic Equations **8.** Evaluate logarithmic expressions **9.** Use the properties of logarithms to evaluate expressions **10.** Perform transformations on the graph of the logarithmic function **11.** Use the change of base theorem to evaluate logs of different bases **12.** Use the exponential and logarithmic properties to solve logarithmic equations **13.** Explore financial applications of exponential functions Activity: Compound Interest	

	Activity: Modeling with Exponential and Logarithmic Equations Functions: Guess the Power Activity: Bank Account Activity: Why Does the Rule of 72 Work? NOTE: Honors level students are expected to work on additional rigorous, challenging problems, proofs and application of concepts/skills as part of the course. Increased pace of instruction will occur.	

DIFFERENTIATION ACTIVITIES:
Teacher directed differentiated instructional projects and activities are ongoing and based on student need.

ENRICHMENT:	• Activity: Are Colleges Still Affordable? • Hazards of Heavy Metal: An Investigation Using Exponential Models • Functions • Project: A Graphical Approach to Compound Interest • Logarithmic Scale • Web-based Math Resources • Small group instruction • Teacher generated/differentiated instruction enrichment and activities • Supporting the range of learners as per teacher manual • Encourage and support learners in explaining how they applied their skills during mathematical tasks • Precalculus teacher's resources and materials	**REMEDIATION:**	• The Inverse of a Function • Graphing the Inverse of a Function • Logarithm Combination Rules • Solving Exponential Equations with Logs #1 • Solving Exponential Equations with Logs (cont.) #2 • Compound Interest #1 • Compound Interest #2 • Simplifying and Solving Logarithms • Continuous Growth and Radioactive Decay

RESOURCES:

- Precalculus: Graphing, Numerical, Algebraic, 7th ed., Demana et. Al., ©2007
- http://www.sosmath.com/index/html
- www.algebrahelp.com
- www.coolmath.com
- www.mathleague.com
- www.interactmath.com

POCONO MOUNTAIN SCHOOL DISTRICT CURRICULUM

Honors Functions	Unit 4:	Trigonometric Functions	Time Frame:	35 - 38 days

NATIONAL COMMON CORE STANDARDS:

Extend the properties of exponents to rational exponents

- **N.RN.1** Explain how the definition of the meaning of rational exponents follows from extending the properties of integer exponents to those values, allowing for a notation for radicals in terms of rational exponents. *For example, we define* $(5^{1/3})^3$ to hold, so $(5^{1/3})^3$ *must equal 5.*

Reason quantitatively and use units to solve problems

- **N.Q.1** Use units as a way to understand problems and to guide the solution of multi-step problems; choose and interpret units consistently in formulas; choose and interpret the scale and the origin in graphs and data displays.
- **N.Q.2** Define appropriate quantities for the purpose of descriptive modeling.
- **N.Q.3** Choose a level of accuracy appropriate to limitations on measurement when reporting quantities.

Perform arithmetic operations on polynomials

- **A.APR.1** Understand that polynomials form a system analogous to the integers, namely, they are closed under the operations of addition, subtraction, and multiplication; add, subtract, and multiply polynomials.

Understand the concept of a function and use function notation

- **F.IF.1** Understand that a function from one set (called the domain) to another set (called the range) assigns to each element of the domain exactly one element of the range. If *f* is a function and x is an element of its domain, then *f(x)* denotes the output of *f* corresponding to the input x. The graph of *f* is the graph of the equation *y = fx*.
- **F.IF.2** Use function notation, evaluate functions for inputs in their domains, and interpret statements that use function notation in terms of a context.
- **F.IF.3** Recognize that sequences are functions, sometimes defined recursively, whose domain is a subset of the integers. For example, the Fibonacci sequence is defined recursively by f(0) = f(1) = 1, f(n+1) = f(n) + f(n-1) for n ≥ 1.

Build new functions from existing functions

- **F.BF.3** Identify the effect on the graph of replacing $f(x)$ *by* $f(x) + k, kf(x), f(kx),$ *and* $f(x+k)$ for specific values of *k* (both positive and negative); find the value of *k* given the graphs. Experiment with cases and illustrate an explanation of the effects on the graph using technology. *Include recognizing even and odd functions from their graphs and algebraic expressions for them.*
- **F.BF.4** Find inverse functions.
- **F.BF.4.a** Solve an equation of the form $f(x) = c$ for a simple function f that has an inverse and write an expression for the inverse. *For example,* $f(x) = 2x^3$ *or* $f(x) = (x+1)/(x-1)$ *for* $x \neq 1$.
- **F.*BF.4.b*** *(+)* Verify by composition that one function is the inverse of another.
- **F.BF.4.c** (+) Read values of an inverse function from a graph or a table, given that the function has an inverse.

MATHEMATICAL PRACTICES:

1. Make sense of problems and persevere in solving them.
2. Reason abstractly and quantitatively.
3. Construct viable arguments and critique the reasoning of others.
4. Model with mathematics.
5. Use appropriate tools strategically.
6. Attend to precision.
7. Look for and make use of structure.
8. Look for and express regularity in repeated reasoning.

- **F.BF.4.d** (+) Produce an invertible function from a non-invertible function by restricting the domain.
- **F.BF.5** (+) Understand the inverse relationship between exponents and logarithms and use this relationship to solve problems involving logarithms and exponents.

Construct and compare linear, quadratic, and exponential models and solve problems

- **F.LE.1** Distinguish between situations that can be modeled with linear functions and with exponential functions.
- **F.LE.1.a** Prove that linear functions grow by equal differences over equal intervals, and that exponential functions grow by equal factors over equal intervals.
- **F.LE.1.b** Recognize situations in which one quantity changes at a constant rate per unit interval relative to another.
- **F.LE.1.c** Recognize situations in which a quantity grows or decays by a constant percent rate per unit interval relative to another.
- **F.LE.2** Construct linear and exponential functions, including arithmetic and geometric sequences, given a graph, a description of a relationship, or two input-output pairs (include reading these from a table).
- **F.LE.3** Observe using graphs and tables that a quantity increasing exponentially eventually exceeds a quantity increasing linearly, quadratically, or (more generally) as a polynomial function.

Interpret expressions for functions in terms of the situation they model

- **F.LE.5** Interpret the parameters in a linear or exponential function in terms of a context.

Extend the domain of trigonometric functions using the unit circle

- **F.TF.1** Understand radian measure of an angle as the length of the arc on the unit circle subtended by the angle.
- **F.TF.2** Explain how the unit circle in the coordinate plane enables the extension of trigonometric functions to all real numbers, interpreted as radian measures of angles traversed counterclockwise around the unit circle.
- **F.TF.3** (+) Use special triangles to determine geometrically the values of sine, cosine, tangent for π-x, π+x, and 2π-x in terms of their values for x, where x is any real number.
- **F.TF.4** (+) Use the unit circle to explain symmetry (odd and even) and periodicity of trigonometric functions.

Model periodic phenomena with trigonometric functions

- **F.TF.5** Choose trigonometric functions to model periodic phenomena with specified amplitude, frequency, and midline.
- **F.TF.6** (+) Understand that restricting a trigonometric function to a domain on which it is always increasing or always decreasing allows its inverse to be constructed.
- **F.TF.7** (+) Use inverse functions to solve trigonometric equations that arise in modeling contexts; evaluate the solutions using technology, and interpret them in terms of the context.

Prove and apply trigonometric identities

- **F.TF.8** Prove the Pythagorean identity $\sin^2(\theta) + \cos^2(\theta) = 1$ and use it to find $\sin(\theta)$, $\cos(\theta)$, or $\tan(\theta)$ given $\sin(\theta)$, $\cos(\theta)$, or $\tan(\theta)$, and the quadrant of the angle.
- **F.TF.9** (+)Prove the addition and subtraction formulas for sine, cosine, and tangent and use them to solve problems.

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Define trigonometric ratios and solve problems involving right triangles

- **G.SRT.6** Understand that by similarity, side ratios in right triangles are properties of the angles in the triangle, leading to definitions of trigonometric ratios for acute angles.
- **G.SRT.7** Explain and use the relationship between the sine and cosine of complementary angles.
- **G.SRT.8** Use trigonometric ratios and the Pythagorean Theorem to solve right triangles in applies problems.

Apply trigonometry to general triangles

- **G.SRT.9** (+)Derive the formula A = ½ ab sin© for the area of a triangle by drawing an auxiliary line from a vertex perpendicular to the opposite side.
- **G.SRT.10** (+)Prove the Laws of Sines and Cosines and use them to solve problems.
- **G.SRT.11** (+)Understand and apply the Law of Sines and the Law of cosines to find unknown measurements in right and non-right triangles (e.g., surveying problems, resultant forces).

ESSENTIAL QUESTIONS	VOCABULARY	ASSESSMENT
1. How would you determine and use central angle measure in radians and degrees? 2. How would you determine the trigonometric functions of an acute angle with respect to a right triangle? 3. Investigate the trigonometric functions with respect to the unit circle. 4. How would you investigate the graphs of the six trigonometric functions? 5. Show how to graph composite functions involving trigonometric functions. 6. Demonstrate how to relate the concept of inverse functions to trigonometric functions. 7. How would you apply concepts of trigonometry to real world situations?	• Radians • Unit Circle • Degrees • Trigonometric functions • Sine • Cosine • Tangent • Period • Composition of functions	• Observation and questioning • Presentations and discussions • Projects and investigations • Mathematical writing • Homework • Quizzes • Tests

UNIT OF INSTRUCTION: Trigonometric Functions	PA CORE STANDARDS	
	CC.2.1.HS.F.1 Apply and extend the properties of exponents to solve problems with rational exponents. **CC.2.1.HS.F.2** Apply properties of rational and irrational numbers to solve real world or mathematical problems. **CC.2.1.HS.F.3** Apply quantitative reasoning to choose and interpret units and scales in formulas, graphs and data displays. **CC.2.1.HS.F.4** Use units as a way to understand problems and to guide the solution of multi-step problems. **CC.2.1.HS.F.5** Choose a level of accuracy appropriate to limitations on measurement when reporting quantities. **CC.2.1.HS.F.6** Extend the knowledge of arithmetic operations and apply to complex numbers. **CC.2.1.HS.F.7** Apply concepts of complex numbers in polynomial identities and quadratic equations. **CC.2.2.HS.D.2** Write expressions in equivalent forms to solve problems. **CC.2.2.HS.D.3** Extend the knowledge of arithmetic operations and apply to polynomials. **CC.2.2.HS.D.4** Understand the relationship between zeros and factors of polynomials to make generalizations about functions and their graphs. **CC.2.2.HS.D.5** Use polynomial identities to solve problems. **CC.2.2.HS.D.6** Extend the knowledge of rational functions to rewrite in equivalent forms. **CC.2.2.HS.D.9** Use reasoning to solve equations and justify the solution method. **CC.2.2.HS.D.10** Represent, solve and interpret equations/inequalities and systems of equations/inequalities algebraically and graphically. **CC.2.2.HS.C.1** Use the concept and notation of functions to interpret and apply them in terms of their context. **CC.2.2.HS.C.2** Graph and analyze functions and use their properties to make connections between the different representations. **CC.2.2.HS.C.4** Interpret the effects transformations have on functions and find the inverses of functions. **CC.2.2.HS.C.5** Construct and compare linear, quadratic and exponential models to solve problems. **CC.2.2.HS.C.6** Interpret functions in terms of the situation they model. **CC.2.2.HS.C.7** Apply radian measure of an angle and the unit circle to analyze the trigonometric functions. **CC.2.2.HS.C.8** Choose trigonometric functions to model periodic phenomena and describe the properties of the graphs. **CC.2.2.HS.C.9** Prove the Pythagorean identity and use it to calculate trigonometric ratios. **CC.2.3.HS.A.1** Use geometric figures and their properties to represent transformations in the plane. **CC.2.3.HS.A.2** Apply rigid transformations to determine and explain congruence. **CC.2.3.HS.A.3** Verify and apply geometric theorems as they relate to geometric figures. **CC.2.3.HS.A.4** Apply the concept of congruence to create geometric constructions. **CC.2.3.HS.A.7** Apply trigonometric ratios to solve problems involving right triangles. **CC.2.3.HS.A.8** Apply geometric theorems to verify properties of circles. **CC.2.3.HS.A.9** Extend the concept of similarity to determine arc lengths and areas of sectors of circle. **Essential Understandings/Learning Activities:** 1. Convert between degrees and radians 2. Define degree and radian measure of an angle 3. Find circular arc length in degrees and radians	

4. Use angular and linear speed to solve practical problems
5. Define the six trigonometric functions of an acute angle

Activity: Some Mnemonics to Remember Your Trig Ratios

6. Use the special right triangles to evaluate the value of the trigonometric functions for 30, 45, and 60 degrees

Activity: Trig Cut Ups

7. Use one trigonometric ratio to find the remaining ones
8. Use a calculator to find the values of trigonometric functions
9. Use trigonometric functions to find the sides of right triangles
10. Explore co-terminal angles
11. Investigate first quadrant trigonometry
12. Use reference triangles to evaluate the trigonometric functions of any angle
13. Determine the trigonometric functions for the quadrantile angles
14. Explore the unit circle and the wrapping function
15. Find trigonometric functions of real numbers
16. Investigate the concept of periodicity
17. Investigate the 16-point unit circle

Activity: Radian, the Snowman

Activity: Radian Walk

18. Investigate the characteristics of the sine and cosine functions

Activity: Sine Cosine Game

19. Explore the transformations of the sine function
20. Determine the amplitude, period frequency and phase shift of a sinusoid
21. Investigate the characteristics of the tangent
22. Cotangent, secant and cosecant graphs
23. Investigate the result of combining trigonometric and algebraic functions
24. Determine when a composite function is periodic
25. Explore sums and differences
26. Determine whether or not a function is a sinusoid
27. Investigate the domain and range of the inverse functions
28. Evaluate inverse functions with and without a calculator
29. Evaluate compositions of trigonometric and inverse trigonometric functions
30. Solve right triangles
31. Apply right triangle trigonometry to real world situations
32. Solve trigonometric equations and inequalities algebraically and graphically
33. Use trigonometric functions to determine the angle between lines
34. Use angle of depression and the angle of elevation in application problems
35. Solve trigonometric equations and inequalities algebraically and graphically

NOTE: Honors level students are expected to work on additional rigorous, challenging problems, proofs and application of concepts/skills as part of the course. Increased pace of instruction will occur.

DIFFERENTIATION ACTIVITIES:
Teacher directed differentiated instructional projects and activities are ongoing and based on student need.

ENRICHMENT:

- Project: Fitting a Model to Data
- Web-based Math Resources
- Small group instruction
- Teacher generated/differentiated instruction enrichment and activities
- Supporting the range of learners as per teacher manual
- Encourage and support learners in explaining how they applied their skills during mathematical tasks
- Precalculus teacher's resources and materials

REMEDIATION:

- Manipulating Special Right Triangles
- Trigonometric Ratios
- Evaluating Trigonometric Functions
- Applying Trigonometric Ratios
- Using Trigonometric Ratios to Find Angles
- Trigonometric Ratios
- Angles Greater than 360 Degrees
- Converting Angle Measurements
- Manipulating Properties of Sin and Cosine
- Graphing Sine and Cosine Functions
- Graphing the Sine and Cosine Functions (cont)
- Graphing y =a sin x or y = a cos x
- Graphing y = c + sin x or y = c + cos x
- Graphing y = sin bx or y = cos bx

RESOURCES:

- Precalculus: Graphing, Numerical, Algebraic, 7th ed., Demana et. Al., ©2007
- http://www.sosmath.com/index/html
- www.algebrahelp.com
- www.coolmath.com
- www.mathleague.com
- www.interactmath.com

POCONO MOUNTAIN SCHOOL DISTRICT CURRICULUM

Honors Functions	Unit 5:	Analytic Trigonometry	Time Frame:	30 - 33 days

NATIONAL COMMON CORE STANDARDS:

Reason quantitatively and use units to solve problems

- **N.Q.1** Use units as a way to understand problems and to guide the solution of multi-step problems; choose and interpret units consistently in formulas; choose and interpret the scale and the origin in graphs and data displays.
- **N.Q.2** Define appropriate quantities for the purpose of descriptive modeling.
- **N.Q.3** Choose a level of accuracy appropriate to limitations on measurement when reporting quantities.

Interpret the structure of expressions

- **A.SSE.1** Interpret expressions that represent a quantity in terms of its context.
- **A.SSE.1.a** Interpret parts of an expression, such as terms, factors, and coefficients.
- **A.SSE.1.b** Interpret complicated expressions by viewing one or more of their parts as a single entity. *For example, interpret $P(1+r)^n$ as the product of P and a factor not depending on P.*

Write expressions in equivalent forms to solve problems

- **A.SSE.3** Choose and produce an equivalent form of an expression to reveal and explain properties of the quantity represented by the expression.
- **A.SSE.3.a** Factor a quadratic expression to reveal the zeros of the function it defines.
- **A.SSE.3.b** Complete the square in a quadratic expression to reveal the maximum or minimum value of the function it defines.

Perform arithmetic operations on polynomials

- **A.APR.1** Understand that polynomials form a system analogous to the integers, namely, they are closed under the operations of addition, subtraction, and multiplication; add, subtract, and multiply polynomials.

Understand solving equations as a process of reasoning and explain the reasoning

- **A.REI.1** Explain each step in solving a simple equation as following from the equality of numbers asserted at the previous step, starting from the assumption that the original equation has a solution. Construct a viable argument to justify a solution method.
- **A.REI.2** Solve simple rational and radical equations in one variable, and give examples showing how extraneous solutions may arise.

Represent and solve equations and inequalities graphically

- **A.REI.10** Understand that the graph of an equation in two variables is the set of all its solutions plotted in the coordinate plane, often forming a curve (which could be a line).
- **A.REI.11** Explain why the *x-coordinates of the points where the graphs of the equations y = f(x) and y = g(x) intersect are the solutions of the equation f(x) = g(x); find the solutions approximately, e.g., using technology to graph the functions, make tables of values, or find successive approximations. Include cases where f(x) and/or g(x) are linear, polynomial, rational,, absolute value, exponential, and logarithmic functions.*

MATHEMATICAL PRACTICES:

1. Make sense of problems and persevere in solving them.
2. Reason abstractly and quantitatively.
3. Construct viable arguments and critique the reasoning of others.
4. Model with mathematics.
5. Use appropriate tools strategically.
6. Attend to precision.
7. Look for and make use of structure.
8. Look for and express regularity in repeated reasoning.

Understand the concept of a function and use function notation

- **F.IF.1** Understand that a function from one set (called the domain) to another set (called the range) assigns to each element of the domain exactly one element of the range. If *f* is a function and *x* is an element of its domain, then *f(x)* denotes the output of *f* corresponding to the input I x I. The graph of *f* is the graph of the equation *y = f(x)*.
- **F.IF.2** Use function notation, evaluate functions for inputs in their domains, and interpret statements that use function notation in terms of a context.

Interpret functions that arise in applications in terms of the context

- **F.IF.4** For a function that models a relationship between two quantities interpret key features of graphs and tables in terms of the quantities, and sketch graphs showing key features given a verbal description of the relationship. Key features include: *intercepts; intervals where the function is increasing, decreasing, positive, or negative; relative maximums and minimums; symmetries; end behavior; and periodicity.*
- **F.IF.5** Relate the domain of a function to its graph and, where applicable, to the quantitative relationship it describes. *For examples, if the function h(n) gives the number person-hours it takes to assemble n engines in a factory, then the positive integers would be an appropriate domain for the function.*
- **F.IF.6** Calculate and interpret the average rate of change of a function (present symbolically or as a table) over a specified interval. Estimate the rate of change from a graph.

Extend the domain of trigonometric functions using the unit circle

- **F.TF.1** Understand radian measure of an angle as the length of the arc on the unit circle subtended by the angle.
- **F.TF.2** Explain how the unit circle in the coordinate plane enables the extension of trigonometric functions to all real numbers, interpret as radian measures of angles traversed counterclockwise around the unit circle.
- **F.TF.3** (+)Use special triangles to determine geometrically the values of sine, cosine, tangent for π/3, π/4 and π/6, and use the unit circle to express the values of sine, cosine, and tangent for π-x, π+x, and 2π-x in terms of their values for x, where x is any real number.
- **F.TF.4** (+)Use the unit circle to explain symmetry (odd and even) and periodicity of trigonometric functions.

Model periodic phenomena with trigonometric functions

- **F.TF.5** Choose trigonometric functions to model periodic phenomena with specified amplitude, frequency, and middling.
- **F.TF.6** (+)Understand that restricting a trigonometric function to a domain on which it is always increasing or always decreasing allows its inverse to be constructed.
- **F.TF.7** (+)Use inverse functions to solve trigonometric equations that arise in modeling context; evaluate the solutions using technology, and interpret them in terms of the context.

Prove and apply trigonometric identities

- **F.TF.8** Prove the Pythagorean identity $\sin^2(\theta) + \cos^2(\theta) = 1$ and use it to find $\sin(\theta)$, $\cos(\theta)$, or $\tan(\theta)$ given $\sin(\theta)$, $\cos(\theta)$, or $\tan(\theta)$ and the quadrant of the angle.
- **F.TF.9** (+)Prove the addition and subtraction formulas for sine, cosine, and tangent and use them to solve problems.

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Define trigonometric ratios and solve problems involving right triangles

- **G.SRT.6** Understand that by similarity, side ratios in right triangles are properties of the angles in the triangle, leading to definitions of trigonometric ratios for acute angles.
- **G.SRT.7** Explain and use the relationship between the sine and cosine of complementary angles.
- **G.SRT.8** Use trigonometric ratios and the Pythagorean Theorem to solve right triangles in applied problems.

Apply trigonometry to general triangles

- **G.SRT.9** (+)Derive the formula A = ½ ab sin(C) for the area of a triangle by drawing an auxiliary line from a vertex perpendicular to the opposite side.
- **G.SRT.10** (+)Prove the Laws of Sines and Cosines and use them to solve problems.
- **G.SRT.11** (+)Understand and apply the Law of Sines and the Law of Cosines to find unknown measurements in right and non-right triangles (e.g., surveying problems, resultant forces).

Translate between the geometric description and the equations for a conic section

- **G.GPE.1** Derive the equation of a circle of given center and radius using the Pythagorean Theorem; complete the square to find the center and radius of a circle given by an equation.

ESSENTIAL QUESTIONS	VOCABULARY	ASSESSMENT
1. How would you use fundamental trigonometric identities to simplify trigonometric expressions and solve trigonometric equations? 2. Show how to prove a variety of trigonometric identities. 3. Demonstrate how to understand and use the laws of sines and cosines. 4. How would you examine the area of a triangle?	• Trigonometric Identity • Cosecant • Secant • Cotangent • Amplitude • Frequency • Inverse Trigonometric Functions	• Observation and questioning • Presentations and discussions • Projects and investigations • Mathematical writing • Homework • Quizzes • Tests

UNIT OF INSTRUCTION: Analytic Trigonometry	PA CORE STANDARDS	
	CC.2.1.HS.F.1 Apply and extend the properties of exponents to solve problems with rational exponents. **CC.2.1.HS.F.2** Apply properties of rational and irrational numbers to solve real world or mathematical problems. **CC.2.1.HS.F.3** Apply quantitative reasoning to choose and interpret units and scales in formulas, graphs and data displays. **CC.2.1.HS.F.4** Use units as a way to understand problems and to guide the solution of multi-step problems. **CC.2.2.HS.D.2** Write expressions in equivalent forms to solve problems. **CC.2.2.HS.D.3** Extend the knowledge of arithmetic operations and apply to polynomials. **CC.2.2.HS.D.4** Understand the relationship between zeros and factors of polynomials to make generalizations about functions and their graphs. **CC.2.2.HS.D.5** Use polynomial identities to solve problems. **CC.2.2.HS.D.8** Apply inverse operations to solve equations or formulas for a given variable. **CC.2.2.HS.D.10** Represent, solve and interpret equations/inequalities and systems of equations/inequalities algebraically and graphically. **CC.2.2.HS.C.1** Use the concept and notation of functions to interpret and apply them in terms of their context. **CC.2.2.HS.C.2** Graph and analyze functions and use their properties to make connections between the different representations. **CC.2.2.HS.C.7** Apply radian measure of an angle and the unit circle to analyze the trigonometric functions. **CC.2.2.HS.C.8** Choose trigonometric functions to model periodic phenomena and describe the properties of the graphs. **CC.2.2.HS.C.9** Prove the Pythagorean identity and use it to calculate trigonometric ratios. **CC.2.3.HS.A.7** Apply trigonometric rations to solve problems involving right triangles. **Essential Understandings/Learning Activities:** 1. Investigate the basic, Pythagorean, cofunction, and odd-even identities Activity: Trigonometry Triangles 2. Simplify trigonometric expressions 3. Solve trigonometric equations 4. Verify trigonometric identities graphically 5. Prove a variety of trigonometric identities analytically Activity: Trig Cut Up (2 versions) 6. Disprove non-identities 7. Apply the cosine, sine, and tangent sum and difference identities 8. Apply the double-angle and half-angle identities Activity: Trigonometric Sum and Difference Identities Activity: I Have … Who Has … Cards Activity: Trig Card Game Activity: Trig Dot-to-Dot Activity: Magic Square 9. Use the laws of sines and cosines to solve triangles	

10. Apply the laws of sines and cosines to real-life situations
11. Develop the formula for the area of a triangle in terms of the sine function
12. Find the area of a triangle using Heron's formula
13. Apply the triangle area formulas to real world problems

NOTE: Honors level students are expected to work on additional rigorous, challenging problems, proofs and application of concepts/skills as part of the course. Increased pace of instruction will occur.

DIFFERENTIATION ACTIVITIES:
Teacher directed differentiated instructional projects and activities are ongoing and based on student need.

ENRICHMENT:

- The Science and Math Connection
- Folding Leg of Card Tables
- Checkpoint
- Modeling the Illumination of the Moon
- Modeling the Motion of a Pendulum
- Evaluating Modeling Solutions

REMEDIATION:

- Law of Sines
- Law of Cosines
- The Law of Sines and the Law of Cosines
- Trig I.D. Problems (2 pages)
- Mixed Problems (3 pages)

RESOURCES:

- Precalculus: Graphing, Numerical, Algebraic, 7th ed., Demana et. Al., ©2007
- www.coolmath.com
- www.mathleague.com
- www.interactmath.com
- http://www.themathpage.com/aTrig/trigonometry.htm

POCONO MOUNTAIN SCHOOL DISTRICT CURRICULUM

Honors Functions	Unit 6:	Vectors, Parametric Equations	Time Frame:	20 days

NATIONAL COMMON CORE STANDARDS:

Represent and model with vector quantities

- **N.VM.1** (+) Recognize vector quantities as having both magnitude and direction. Represent vector quantities by directed line segments, and use appropriate symbols for vectors and their magnitudes (e.g., **v,** |**v**|, ||**v**||, **v**).
- **N.VM.2** (+) Find the components of a vector by subtracting the coordinates of an initial point from the coordinates of a terminal point.
- **N.VM.3** (+) Solve problems involving velocity and other quantities that can be represented by vectors.

Perform operations on vectors

- **N.VM.4** (+) Add and subtract vectors.
- **N.VM.4.a** Add vectors end-to-end, component-wise, and by the parallelogram rule. Understand that the magnitude of a sum of two vectors is typically not the sum of the magnitudes.
- **N.VM.4.b** Given two vectors in magnitude and direction form, determine the magnitude and direction of their *sum*.
- **N.VM.4.c** Understand vector subtraction **v** - **w** as **v** + (-**w**), where –**w** is the additive inverse of **w**, with the same magnitude as **w** and pointing in the opposite direction. Represent vector subtraction graphically by connecting the tips in the appropriate order, and perform vector subtraction component-wise.
- **N.VM.5** (+) Multiply a vector by a scalar.
- **N.VM.5.a** Represent scalar multiplication graphically by scaling vectors and possibly reversing their direction; perform scalar multiplication component-wise, e.g., as c(vx, vy) = (cvx, cvy).
- **N.VM.5.b** Compute the magnitude of a scalar multiple c**v** using ||c**v**|| = |c|**v**. Compute the direction of c**v** knowing that when |c|**v** ≠ 0, the direction of c**v** is either along **v** (for c > 0) or against **v** (for c < 0).

MATHEMATICAL PRACTICES:

1. Make sense of problems and persevere in solving them.
2. Reason abstractly and quantitatively.
3. Construct viable arguments and critique the reasoning of others.
4. Model with mathematics.
5. Use appropriate tools strategically.
6. Attend to precision.
7. Look for and make use of structure.
8. Look for and express regularity in repeated reasoning.

ESSENTIAL QUESTIONS	VOCABULARY	ASSESSMENT
1. Show how you would apply the arithmetic of vectors and use vectors to solve real-world problems. 2. Define parametric equations, graph curves parametrically, and solve application problems using parametric equations.	• Vector • Magnitude and direction • Velocity • Force and work • Parametric equations	• Observation and questioning • Presentations and discussions • Projects and investigations • Mathematical writing • Homework • Quizzes • Tests

POCONO MOUNTAIN SCHOOL DISTRICT CURRICULUM

UNIT OF INSTRUCTION: Vectors, Parametric Equations

PA CORE STANDARDS

CC.2.2.HS.C.2 Graph and analyze functions and use their properties to make connections between the different representations.

Essential Understandings/Learning Activities:

14. Classify quantities as either vector or scalar
15. Use terminology associated with vectors
16. Determine whether or not two vectors are equal
17. Find the component form of a vector
18. Perform vector addition and scalar multiplication

Activity: Adding Vectors Graphically

19. Find the direction angle of a vector
20. Use vectors to represent quantities such as force and velocity
21. Calculate dot products and find the length of vectors
22. Find the angle between vectors
23. Apply vectors to problems involving force and work
24. Graph parametric equations
25. Eliminate the parameter to obtain a rectangular equation in x and y
26. Use a grapher to simulate motion

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DIFFERENTIATION ACTIVITIES:

Teacher directed differentiated instructional projects and activities are ongoing and based on student need.

ENRICHMENT:

- Trigonometry in Automobile Accident Reconstruction
- Vector Equations in Three Dimension
- Project: Precalculus with Limits
- Project: Tangent Lines to Sine Curves

REMEDIATION:

- Describing Vectors
- Adding Vectors Graphically
- Vector Addition and Scalar Multiplication
- Resolving Vectors
- Describing Vectors from x and y Components
- Adding Vectors Algebraically
- Vector Dot Product
- The Angle Between Two Vectors

RESOURCES:	
	• Precalculus: Graphing, Numerical, Algebraic, 7th ed., Demana et. Al., ©2007 • http://www.sosmath.com/index.html • www.coolmath.com • www.mathleague.com • www.interactmath.com • http://www.themathpage.com/aTrig/trigonometry.htm